

$$\textcircled{\beta} \begin{cases} u_0 = 3 \\ u_{n+1} = \frac{1}{3}u_n + \frac{2}{3} \end{cases}$$

$$\begin{aligned} 1 \text{ a) } u_1 &= \frac{1}{3}u_0 + \frac{2}{3} = 1 + \frac{2}{3} = \frac{5}{3} \\ u_2 &= \frac{1}{3}u_1 + \frac{2}{3} = \frac{5}{9} + \frac{6}{9} = \frac{11}{9} \\ u_3 &= \frac{1}{3}u_2 + \frac{2}{3} = \frac{11}{27} + \frac{18}{27} = \frac{29}{27} \end{aligned}$$

(u_n) n'est pas arithmétique car

$$u_1 - u_0 = \frac{5}{3} - 3 = -\frac{4}{3} \neq u_2 - u_1 = \frac{11}{9} - \frac{5}{3} = \frac{11}{9} - \frac{15}{9} = -\frac{4}{9}$$

(u_n) n'est pas géométrique car

$$\frac{u_1}{u_0} = \frac{5}{3} \cdot \frac{1}{3} = \frac{5}{9} \neq \frac{u_2}{u_1} = \frac{11}{9} \cdot \frac{3}{5} = \frac{11}{15}$$

$$b) l = \frac{1}{3}l + \frac{2}{3}$$

$$\frac{2}{3}l = \frac{2}{3}$$

$$l = 1$$

$$2^\circ v_n = u_n - 1$$

$$a) \frac{v_{n+1}}{v_n} = \frac{u_{n+1} - 1}{u_n - 1} = \frac{\frac{1}{3}u_n + \frac{2}{3} - 1}{u_n - 1} = \frac{\frac{1}{3}u_n - \frac{1}{3}}{u_n - 1} = \frac{\frac{1}{3}(u_n - 1)}{u_n - 1} = \frac{1}{3}$$

(v_n) est géométrique de raison $\frac{1}{3}$ et de premier terme $v_0 = u_0 - 1 = 3 - 1 = 2$

$$v_n = 2 \cdot \left(\frac{1}{3}\right)^n$$

$$u_n = 1 + v_n = 1 + 2 \cdot \left(\frac{1}{3}\right)^n$$

$$1) \lim_{n \rightarrow +\infty} u_n = \lim_{n \rightarrow +\infty} 1 + 2 \cdot \left(\frac{1}{3}\right)^n = 1 \text{ vérifié!}$$