Exercice 1

① $g(-1) = 2(-1)^3 + 4(-1)^2 - 10(-1) - 12 = 0$

$g(2) = 2 \cdot 2^3 + 4 \cdot 2^2 - 10 \cdot 2 - 12 = 0$

On en déduit que -1 et 2 sont les racines de $g(x)$

et que $g(x) = (x+1)(x-2)(ax+b)$.

② $g(x) = 2x^3 + 4x^2 - 10x - 12 = (x+1)(x-2)(ax+b)$

$= (x^2 - x - 2)(ax+b)$

donc $a \cdot 1 = 2 \rightarrow a = 2$

et $-2 \cdot b = -12 \rightarrow b = 6$

donc $g(x) = (x+1)(x-2)(2x+6)$.

ou $g(x) = 2(x+1)(x-2)(x+3)$.

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③ $h(x) = x^4 - 10x^2 + 9$ on pose $t = x^2$

d'où $t^2 - 10t + 9 = 0$

$\Delta = (-10)^2 - 4 \cdot 1 \cdot 9$

$\Delta = 64$

$t_1 = \frac{10+8}{2} = 9$

$t_2 = \frac{10-8}{2} = 1$

on en déduit que $t^2 - 10t + 9 = (t-9)(t-1)$

donc $h(x) = (x^2 - 9)(x^2 - 1)$

$h(x) = (x-3)(x+3)(x-1)(x+1)$.

Étant donné que $f(x) = \frac{g(x)}{h(x)}$, $\mathcal{D}_f$: $h(x) \neq 0$

donc $\mathcal{D}_f = \mathbb{R} \setminus \{-3, -1, 1, 3\}$.

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④ pour tout x de $\mathcal{D}_f$: $f(x) = \frac{g(x)}{h(x)}$

$$\frac{g(x)}{h(x)} = \frac{2\cancel{(x+1)}(x-2)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}(x-1)\cancel{(x+1)}} = \frac{2(x-2)}{(x-3)(x-1)} = \frac{2x-4}{x^2-4x+3}$$

c.q.f.d.

⑤ $f(x) \leq 0$

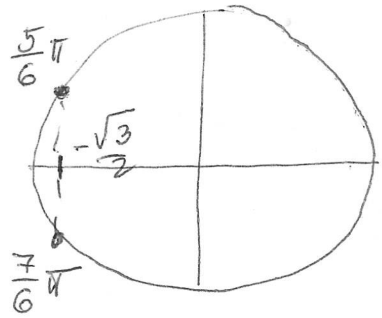
tableau de signes

x	$-\infty$	-3		-1		1		2		3		$+\infty$
$2x-4$		$-$		$-$		$-$		$-$	0	$+$		$+$
$x-3$		$-$		$-$		$-$		$-$		$-$	0	$+$
$x-1$		$-$		$-$		$-$	0	$+$		$+$		$+$
$f(x)$		$-$	‖	$-$	‖	$-$	‖	$+$	0	$-$	‖	$+$

$S =]-\infty ; -3[\cup]-3, -1[\cup]-1, 1[\cup [2, 3[$.

Ex. 2.

1° $\cos \underbrace{2x}_{t} = -\frac{\sqrt{3}}{2}$



$\cos t = -\frac{\sqrt{3}}{2}$

$t = \frac{5}{6}\pi + 2k\pi$ ou

$t = \frac{7}{6}\pi + 2k\pi \; ; \; k \in \mathbb{Z}$

donc $x = \frac{5}{12}\pi + k\pi$ ou $x = \frac{7}{12}\pi + k\pi \; , \; k \in \mathbb{Z}$

Solutions dans $[0, 2\pi[$: $S = \left\{ \frac{5}{12}\pi ; \frac{17}{12}\pi ; \frac{7}{12}\pi ; \frac{19}{12}\pi \right\}$

2° $2\cos^2 x - 3\cos x + 1 = 0$ on pose : $\cos x = t$

$2t^2 - 3t + 1 = 0$

$\Delta = 9 - 4 \cdot 2 \cdot 1 = 1$

$t_{1/2} = \frac{3 \pm 1}{4} = \begin{cases} 1 \\ \frac{1}{2} \end{cases}$

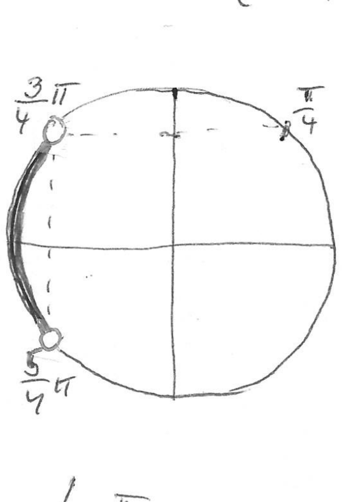
$\cos x = 1$ ssi $x = 0 + 2k\pi$

$\cos x = \frac{1}{2}$ ssi $x = \frac{\pi}{3} + 2k\pi$

ou $x = \frac{-\pi}{3} + 2k\pi$

Solutions dans $[0; 2\pi[$: $S = \left\{ 0 ; \frac{\pi}{3} ; \frac{5}{3}\pi \right\}$

3° $\sin\left(\underbrace{x - \frac{\pi}{2}}_{t}\right) > \frac{\sqrt{2}}{2}$



$\sin t > \frac{\sqrt{2}}{2}$

$\frac{\pi}{4} < t < \frac{3}{4}\pi$

$\frac{\pi}{4} < x - \frac{\pi}{2} < \frac{3}{4}\pi \quad / + \frac{\pi}{2}$

$\frac{3}{4}\pi < x < \frac{5}{4}\pi$

$S = \left] \frac{3}{4}\pi ; \frac{5}{4}\pi \right[$

Exercice n°3

$f(x) = 2\cdot\sin\left(x+\frac{\pi}{3}\right)$

1) Pour tout $x \in D_f$ $\quad -1 \leq \sin\left(x+\frac{\pi}{3}\right) \leq 1 \quad /\cdot 2$

$$-2 \leq 2\cdot\sin\left(x+\frac{\pi}{3}\right) \leq 2$$

$$\underline{\underline{-2 \leq f(x) \leq 2}}$$

2) $f\left(\frac{\pi}{3}\right) = 2\cdot\sin\left(\frac{\pi}{3}+\frac{\pi}{3}\right) = 2\sin\left(\frac{2\pi}{3}\right) = 2\cdot\frac{\sqrt{3}}{2} = \underline{\underline{\sqrt{3}}}$

$f\left(-\frac{\pi}{6}\right) = 2\cdot\sin\left(-\frac{\pi}{6}+\frac{2\pi}{6}\right) = 2\sin\left(\frac{\pi}{6}\right) = 2\cdot\frac{1}{2} = \underline{\underline{1}}$

$f(\pi) = 2\cdot\sin\left(\pi+\frac{\pi}{3}\right) = 2\cdot\sin\left(\frac{4\pi}{3}\right) = 2\cdot\left(-\frac{\sqrt{3}}{2}\right) = \underline{\underline{-\sqrt{3}}}$

$f(-\pi) = 2\cdot\sin\left(-\pi+\frac{\pi}{3}\right) = 2\cdot\sin\left(-\frac{2\pi}{3}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = \underline{\underline{-\sqrt{3}}}$

3) avec l'axe des abscisses : $\quad 0 = 2\cdot\sin\left(x+\frac{\pi}{3}\right) \quad /:2$

$$\sin\left(x+\frac{\pi}{3}\right) = 0$$

$$y = x+\frac{\pi}{3}$$

$$\sin y = 0 \iff y_1 = 0, \quad y_2 = \pi$$

$$y_1 = x_1+\frac{\pi}{3}$$

$$0-\frac{\pi}{3} = x_1 \qquad \underline{x_1 = -\frac{\pi}{3}}$$

$$y_2 = x_2+\frac{\pi}{3}$$

$$\pi-\frac{\pi}{3} = x_2 \qquad \underline{x_2 = \frac{2\pi}{3}}$$

$\underline{\underline{P\left(-\frac{\pi}{3};0\right),\ Q\left(\frac{2\pi}{3};0\right)}}$

avec l'axe des ordonnées : $\quad y = 2\cdot\sin\left(0+\frac{\pi}{3}\right)$

$$y = \sqrt{3}$$

$\underline{\underline{R(0;\sqrt{3})}}$

$f(x) = 2\sin\left(x + \frac{\pi}{3}\right)$