

## Devoir 11:

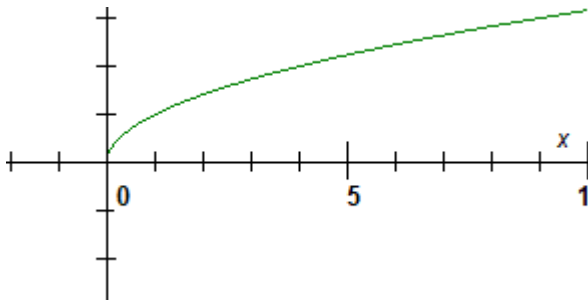
### Fonctions racines n-ième(odmocninné funkce)

Fonction racine n-ième se note  $f(x) = \sqrt[n]{x}$ ,  $n \in \mathbb{N} - \{0\}$ .

$$\sqrt[n]{x} = y \Leftrightarrow x = y^n$$

On utilise aussi la notation :  $\sqrt[n]{x} = x^{\frac{1}{n}}$

Représentation graphique:



### Règles du calcul:

$$n, m \in \mathbb{N}, \quad a, b \in \mathbb{R}^+$$

- 1)  $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$
- 2)  $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
- 3)  $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a}$
- 4)  $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

ou(en utilisant l'exposant rationnel):

- 1)  $a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = (a \cdot b)^{\frac{1}{n}}$
- 2)  $\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$
- 3)  $\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = a^{\frac{1}{n \cdot m}}$
- 4)  $\left(a^{\frac{1}{n}}\right)^m = a^{\frac{m}{n}}$

**Exercice 1:** Eliminer les racines dans le dénominateur:

a)  $\frac{1}{\sqrt{3}}, \frac{12}{\sqrt{6}}, \frac{\sqrt{3}-\sqrt{14}}{\sqrt{6}}$  (rappel:  $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ )

b)  $\frac{2}{3-\sqrt{5}}, \frac{1}{1+\sqrt{7}}, \frac{2+\sqrt{8}}{6-3\sqrt{2}}, \frac{5-\sqrt{3}}{5+\sqrt{3}}$  (rappel:  $\frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2(3+\sqrt{5})}{9-5} = \frac{2(3+\sqrt{5})}{4} = \frac{3+\sqrt{5}}{2}$ )

c)  $\frac{a+\sqrt{b}}{\sqrt{b}}, \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

Résultats:

a)  $\frac{\sqrt{3}}{3}, 2\sqrt{6}, \frac{1}{2}\sqrt{2} - \frac{1}{3}\sqrt{21}$

b)  $\frac{3+\sqrt{5}}{2}, \frac{\sqrt{7}-1}{6}, \frac{4}{3} + \sqrt{2}, \frac{14-5\sqrt{3}}{11}$

c)  $\frac{a\sqrt{b}+b}{b}, \frac{a+b-2\sqrt{ab}}{a-b}$

**Exercice 2:** Simplifier + conditions:

a)  $\sqrt{x\sqrt{x}}$

b)  $\sqrt{z^3\sqrt{z^2}\sqrt{z}}$

c)  $\sqrt[3]{k^2\sqrt{k}}$

d)  $\sqrt[4]{m} \cdot \sqrt[4]{m^3}$

e)  $\sqrt{y^3\sqrt{\frac{1}{y}}\sqrt[4]{y}}$

f)  $\frac{\sqrt[3]{a\sqrt{a^3}}}{\sqrt{a^3\sqrt{a^2}}}$

Résultats:

a)  $\sqrt[4]{x^3} \quad x \geq 0$

b)  $z^2 \sqrt[8]{z} \quad z \geq 0$

c)  $\sqrt[6]{k^5} \quad k \geq 0$

d)  $m; m \geq 0$

e)  $\sqrt[8]{y^3}; y > 0$

f)  $1; a > 0$

**Exercise 3: Simplifier:**

a)  $\frac{x^{\frac{1}{2}} \cdot x^{\frac{3}{4}}}{x^2}$

b)  $\frac{y^2}{\left(y^{\frac{1}{6}}\right)^2}$

c)  $\frac{\left(x^{\frac{2}{3}}\right)^{\frac{1}{4}} \cdot \left(x^{\frac{1}{6}}\right)^{\frac{9}{4}}}{\left(x^{\frac{7}{2}}\right)^{\frac{5}{6}}}$

d)  $\frac{\left(y^{\frac{1}{2}}\right)^3 \cdot \left(y^2\right)^{\frac{1}{3}}}{y \cdot y^{\frac{2}{3}}}$

**Résultats:**

a)  $x^{-\frac{3}{4}}$

b)  $y^{\frac{5}{3}}$

c)  $x^{-\frac{19}{8}}$

d)  $y^{\frac{1}{2}}$

Nápověda k exercice 2:

$$\text{b) } \sqrt{z^3 \sqrt{z^2} \sqrt{z}} = \sqrt{z^3 \sqrt{z^4 \cdot z}} = \sqrt{z^3 \cdot \sqrt[4]{z^5}} = \sqrt{\sqrt[4]{z^{12}} \cdot z^5} = \sqrt[8]{z^{17}} = \sqrt[8]{z^{16} \cdot z} = z^2 \cdot \sqrt[8]{z}$$

$$\text{ou: } \sqrt{z^3 \sqrt{z^2} \sqrt{z}} = z^{\frac{3}{2}} \cdot \left(z^{\frac{2}{2}}\right)^{\frac{1}{2}} \cdot \left(\left(z^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = z^{\frac{3}{2}} \cdot z^{\frac{1}{2}} \cdot z^{\frac{1}{8}} = z^{\frac{12+4+1}{8}} = z^{\frac{17}{8}} = z^{\frac{16}{8}} \cdot z^{\frac{1}{8}} = z^2 \cdot \sqrt[8]{z}$$

$$\text{e) } \sqrt{y^3 \sqrt{\frac{1}{y}} \sqrt[4]{y}} = \sqrt{y \cdot \sqrt[3]{y^{-1}} \cdot \sqrt[4]{y}} = \sqrt{y \cdot \sqrt[4]{y^{-4}} \cdot y} = \sqrt{y \cdot \sqrt[12]{y^{-3}}} = \sqrt{\sqrt[12]{y^{12}} \cdot y^{-3}} = \sqrt[24]{y^9} = y^{\frac{9}{24}} = y^{\frac{3}{8}} = \sqrt[8]{y^3}$$

$$\text{ou: } \sqrt{y^3 \sqrt{\frac{1}{y}} \sqrt[4]{y}} = y^{\frac{3}{2}} \cdot \left(y^{-1}\right)^{\frac{1}{3}} \cdot \left(\left(y^{\frac{1}{4}}\right)^{\frac{1}{3}}\right)^{\frac{1}{2}} = y^{\frac{3}{2}} \cdot y^{-\frac{1}{6}} \cdot y^{\frac{1}{24}} = y^{\frac{12-4+1}{24}} = y^{\frac{9}{24}} = y^{\frac{3}{8}} = \sqrt[8]{y^3}$$