Devoir 11:

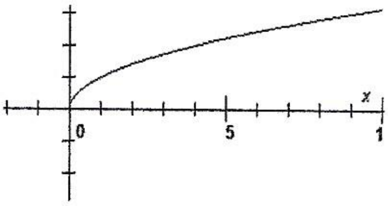
Fonctions racines n-ième(odmocninné funkce)

Fonction racine n-ième se note $f(x) = \sqrt[n]{x}\ ,\ n \in N-(0)$.

$\sqrt[n]{x} = y \ \Leftrightarrow\ x = y^n$

On utilise aussi la notation : $\sqrt[n]{x} = x^{\frac{1}{n}}$

Représentation graphique:



Règles du calcul:

$n, m \in N, \qquad a, b \in R^+$

1) $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$
2) $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
3) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a}$
4) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

ou(en utilisant l'exposent rationel):

1) $a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = (a \cdot b)^{\frac{1}{n}}$
2) $\frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$
3) $\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = a^{\frac{1}{n \cdot m}}$
4) $\left(a^{\frac{1}{n}}\right)^m = a^{\frac{m}{n}}$

$\sqrt[n]{a^n} = a$

$\sqrt[s]{a^n} = a^{\frac{n}{s}}$

Exercice 1 Eliminer les racines dans le dénominateur:

a) $\frac{1}{\sqrt{3}}, \frac{12}{\sqrt{6}}, \frac{\sqrt{3}-\sqrt{14}}{\sqrt{6}}$ (rappel: $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$)

b) $\frac{2}{3-\sqrt{5}}, \frac{1}{1+\sqrt{7}}, \frac{2+\sqrt{8}}{6-3\sqrt{2}}, \frac{5-\sqrt{3}}{5+\sqrt{3}}$ (rappel: $\frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2(3+\sqrt{5})}{9-5} = \frac{2(3+\sqrt{5})}{4} = \frac{3+\sqrt{5}}{2}$)

c) $\frac{a+\sqrt{b}}{\sqrt{b}}, \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}$

Résultats:

a) $\frac{\sqrt{3}}{3}, 2\sqrt{6}, \frac{1}{2}\sqrt{2} - \frac{1}{3}\sqrt{21}$

b) $\frac{3+\sqrt{5}}{2}, \frac{\sqrt{7}-1}{6}, \frac{4}{3}+\sqrt{2}, \frac{14-5\sqrt{3}}{11}$

c) $\frac{a\sqrt{b}+b}{b}, \frac{a+b-2\sqrt{ab}}{a-b}$

a)
- $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
- $\frac{12}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{12\sqrt{6}}{6} = 2\sqrt{6}$
- $\frac{\sqrt{3}-\sqrt{14}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{(\sqrt{3}-\sqrt{14}) \cdot \sqrt{6}}{6} = \frac{\sqrt{18}-\sqrt{84}}{6} = \frac{3\sqrt{2}-2\sqrt{21}}{6} = \frac{\sqrt{2}}{2} - \frac{\sqrt{21}}{3}$

b)
- $\frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{6+2\sqrt{5}}{9-5} = \frac{6+2\sqrt{5}}{4} = \frac{2 \cdot (3+\sqrt{5})}{4} = \frac{3+\sqrt{5}}{2}$
- $\frac{1}{1+\sqrt{7}} \cdot \frac{1-\sqrt{7}}{1-\sqrt{7}} = \frac{1-\sqrt{7}}{1-7} = -\frac{1-\sqrt{7}}{6} = \frac{-1+\sqrt{7}}{6}$
- $\frac{2+\sqrt{8}}{6-3\sqrt{2}} \cdot \frac{6+3\sqrt{2}}{6+3\sqrt{2}} = \frac{12+6\sqrt{8}+6\sqrt{2}+3\sqrt{16}}{36-9 \cdot 2} = \frac{12+12\sqrt{2}+6\sqrt{2}+12}{18} = \frac{24+18\sqrt{2}}{18} = \frac{6 \cdot (4+3\sqrt{2})}{18}$

  $= \frac{4+3\sqrt{2}}{3} = \frac{4}{3} + \sqrt{2}$
- $\frac{5-\sqrt{3}}{5+\sqrt{3}} \cdot \frac{5-\sqrt{3}}{5-\sqrt{3}} = \frac{25-5\sqrt{3}-5\sqrt{3}+3}{25-3} = \frac{28-10\sqrt{3}}{22} = \frac{2(14-5\sqrt{3})}{22} = \frac{14-5\sqrt{3}}{11}$

c)
- $\frac{a+\sqrt{b}}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a\sqrt{b}+b}{b}$
- $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}} \cdot \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{a-\sqrt{ab}-\sqrt{ab}+b}{a-b} = \frac{a+b-2\sqrt{ab}}{a-b}$