Ex. 1:

1. M_{AB} .... K milieu du segment [AB]

$\vec{AB} = B - A = (-5, -3)$

$K = \frac{A+B}{2} = \left(-\frac{1}{2}, \frac{3}{2}\right)$

$-5x - 3y + c = 0$
$-5\cdot\left(-\frac{1}{2}\right) - 3\cdot\left(\frac{3}{2}\right) + c = 0$
$\frac{5}{2} - \frac{9}{2} + c = 0$
$c = 2$

$\underline{-5x - 3y + 2 = 0}$

M_{BC} ... L milieu du segment [BC]

$\vec{BC} = C - B = (4, -4)$

$L = \frac{C+B}{2} = (-1, -2)$

$4x - 4y + c = 0$
$4\cdot(-1) - 4\cdot(-2) + c = 0$
$-4 + 8 + c = 0$
$c = -4$

$4x - 4y - 4 = 0 \quad /:4$
$\underline{x - y - 1 = 0}$

2. $-5x - 3y + 2 = 0$
$x - y - 1 = 0 \quad /\cdot 5$

$-5x - 3y + 2 = 0$
$5x - 5y - 5 = 0$

$0 - 8y - 3 = 0$
$-8y = 3$
$8y = -3$
$y = -\frac{3}{8}$

$x - \left(-\frac{3}{8}\right) - 1 = 0$
$x + \frac{3}{8} - \frac{8}{8} = 0$
$x = \frac{5}{8}$

$\underline{S\left(\frac{5}{8}, -\frac{3}{8}\right)}$

3.
$\vec{BA} = A - B = (5, 3)$ $\qquad BA = \sqrt{34}$
$\vec{BC} = C - B = (4, -4)$ $\qquad BC = \sqrt{32}$

$\cos \widehat{ABC} = \frac{5\cdot 4 + 3\cdot(-4)}{\sqrt{34}\cdot\sqrt{32}} = \frac{20-12}{8\sqrt{17}} = \frac{1}{\sqrt{17}}$

$\underline{\beta = 75{,}96^\circ}$

4.
h_A ... la hauteur issue du point A

$\vec{BC} = (4, -4)$

$A \in h_A$: $4x - 4y + c = 0$
$4\cdot 2 - 4\cdot 3 + c = 0$
$8 - 12 + c = 0$
$-4 + c = 0$
$c = 4$

$4x - 4y + 4 = 0 \quad /:4$
$\underline{x - y + 1 = 0}$

h_B ... la hauteur issue du point B

$\vec{AC} = C - A = (-1, -7)$

$B \in h_B$: $-1x - 7y + c = 0$
$-1\cdot(-3) - 7\cdot 0 + c = 0$
$3 - 0 + c = 0$
$c = -3$

$-x - 7y - 3 = 0 \quad /\cdot(-1)$
$\underline{x + 7y + 3 = 0}$

5. $x - y + 1 = 0 \quad /\cdot(-1)$
$x + 7y + 3 = 0$

$-x + y - 1 = 0$
$x + 7y + 3 = 0$

$0 + 8y + 2 = 0$
$8y = -2$
$y = -\frac{2}{8} = -\frac{1}{4}$

$x - \left(-\frac{1}{4}\right) + 1 = 0$
$x + \frac{1}{4} + \frac{4}{4} = 0$
$x + \frac{5}{4} = 0$
$x = -\frac{5}{4}$

$\underline{O\left(-\frac{5}{4}, -\frac{1}{4}\right)}$